MathCastle

Grade 4

Complete course booklet • 8 topics

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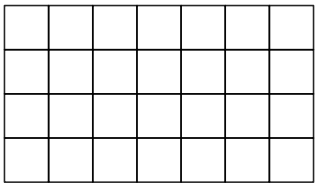
1 Multi-Digit Multiplication

Breaking numbers apart to multiply

1.1 The Area Model

The Big Idea

Split each number into its places and multiply the parts, then add. Drawing it as a rectangle shows why nothing gets missed: the pieces fill the whole area.



each piece is a partial product

Multiplying a big number is really several small multiplications added together. When you split 23 into $20 + 3$ and multiply each piece by 4, you are using the distributive property — you just have not been told its name yet. The area model makes the pieces visible: every part of the rectangle is one partial product, and nothing can be skipped or counted twice.

Finding 23×4

$20 \times 4 = 80$ and $3 \times 4 = 12$. Adding the parts, $80 + 12 = 92$.

Where it goes wrong

Forgetting a partial product when a number has three or four digits, or lining columns up wrongly when adding them. Write each partial product on its own line, in place-value order, and check that you have one for every pair of digits.

1.2 In Plain Terms & More Examples

In plain terms

Break each number into tens and ones, multiply every piece, then add the pieces up — that is the **distributive property**, $a(b + c) = ab + ac$. Nothing gets skipped.

Finding 34×6

$30 \times 6 = 180$ and $4 \times 6 = 24$. Add them: $180 + 24 = 204$.

Finding 52×7

$52 \times 7 = (50 + 2) \times 7$, which equals, which is $7 \times 50 + 7 \times 2$. Add them: $350 + 14 = 364$.

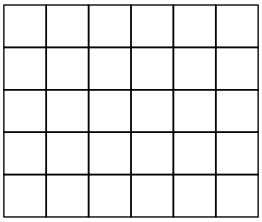
A multi-step problem

A school orders 3 boxes of 24 pencils and 2 boxes of 36 pencils. Step 1: $3 \times 24 = 72$. Step 2: $2 \times 36 = 72$. Step 3: add the parts, $72 + 72 = 144$ pencils in all.

1.3 Two-Digit by Two-Digit

Four partial products

With two two-digit numbers the area model has **four** pieces: tens $\times$ tens, tens $\times$ ones, ones $\times$ tens, ones $\times$ ones. Add all four.



four pieces

Finding 34×26

$30 \times 20 = 600$, $30 \times 6 = 180$, $4 \times 20 = 80$, $4 \times 6 = 24$. Sum: $600 + 180 + 80 + 24 = 884$.

Estimate first

34×26 is about $30 \times 30 = 900$. An answer near 900 is believable; 88 or 8840 is not.

Try it: 47×23

Split both numbers into tens and ones, write the four partial products, and add.

Answer. $40 \times 20 = 800$, $40 \times 3 = 120$, $7 \times 20 = 140$, $7 \times 3 = 21$. Total $800 + 120 + 140 + 21 = \mathbf{1081}$.

1.4 Word Problems

Rows of chairs

A hall has 18 rows with 24 chairs in each. How many chairs? $18 \times 24 = 10 \times 24 + 8 \times 24 = 240 + 192 = 432$ chairs.

Try it: stickers

A sticker book has 36 pages with 15 stickers on each page. How many stickers in all?

Answer. $36 \times 15 = 30 \times 15 + 6 \times 15 = 450 + 90 = \mathbf{540}$ stickers.

1.5 Why It Works

The area model is a proof that the method is correct. A rectangle 23 wide and 4 tall has area 23×4. Cut it into a 20×4 piece and a 3×4 piece: the two pieces exactly fill the rectangle, so their areas $80 + 12$ must add up to the whole, 92. In symbols this is $(20 + 3) \times 4 = 20 \times 4 + 3 \times 4$, the distributive property — the same law that will later let you expand $(x + 3)(x + 4)$.

Reading a proof

Do not read a proof the way you read a rule. Cover the last line, ask what the previous line forces, and only then uncover it. If a step feels like magic, that is the step to work by hand with a concrete number.

2 Long Division

Quotients and remainders

2.1 Sharing What Is Left

The Big Idea

Divide as far as you can in whole groups. Whatever cannot be shared is the **remainder**, and it is always smaller than the divisor.

Division asks how many equal groups you can make. Long division does that one place value at a time: how many hundreds, then how many tens, then how many ones, subtracting what you have used at each step. Whatever is left at the end is the remainder, and it must be smaller than the divisor — otherwise you could have made one more group.

The check

Always: divisor $\times$ quotient $+$ remainder $=$ the original number. If that fails, something went wrong.

Finding $47 \div 5$

$5 \times 9 = 45$, and $47 - 45 = 2$. So the quotient is 9 remainder 2. Check: $5 \times 9 + 2 = 47$.

Where it goes wrong

A remainder that is bigger than the divisor. If you are dividing by 6 and see a remainder of 8, one more group of 6 fits — go back and add 1 to that digit of the quotient.

2.2 In Plain Terms & More Examples

In plain terms

Share into equal groups as far as you can; whatever is left over is the remainder, and it is always smaller than the number you divided by.

Finding $58 \div 6$

$6 \times 9 = 54$, and $58 - 54 = 4$. So 9 remainder 4. Check: $6 \times 9 + 4 = 58$.

Finding $73 \div 8$

$8 \times 9 = 72$, and $73 - 72 = 1$. So 9 remainder 1. Check: $8 \times 9 + 1 = 73$.

2.3 Dividing Three-Digit Numbers

Place by place

Ask how many times the divisor fits into the hundreds, then bring down the tens, then the ones. Each answer digit goes above the digit you just used.

Finding $372 \div 4$

4 into 3: does not fit, so use 37. $4 \times 9 = 36$, remainder 1; bring down the 2 to make 12; $4 \times 3 = 12$, remainder 0. Quotient **93**. Check: $4 \times 93 = 372$.

Zero in the middle

If a step gives 0, WRITE the 0 in the quotient. $618 \div 6 = 103$, not 13.

Try it: $845 \div 5$

Work place by place and check by multiplying back.

Answer. 5 into 8 is 1 remainder 3; $34 \div 5 = 6$ remainder 4; $45 \div 5 = 9$. Quotient **169**. Check: $5 \times 169 = 845$.

2.4 Word Problems

Packing eggs

150 eggs are packed in cartons of 12. $150 \div 12 = 12$ remainder 6: twelve full cartons and 6 eggs left over — so 13 cartons are needed to carry them all.

Try it: buses

A school needs to move 263 students on buses that hold 40. How many buses are needed?

Answer. $263 \div 40 = 6$ remainder 23. Six buses carry 240; the remaining 23 students need a seventh bus: **7** buses. (The remainder means "one more", not "ignore it".)

2.5 Why It Works

Every division fits one equation: dividend = divisor × quotient + remainder, with $0 \leq$ remainder $<$ divisor. That is why you can always check: for $95 \div 4$ we get 23 remainder 3, and $4 \times 23 + 3 = 95$ ✓. The step-by-step subtraction is just this equation being built one place value at a time, and the check works for every division you will ever do.

Reading a proof

Do not read a proof the way you read a rule. Cover the last line, ask what the previous line forces, and only then uncover it. If a step feels like magic, that is the step to work by hand with a concrete number.

3 Equivalent Fractions

Simplifying to lowest terms

3.1 Lowest Terms

The Big Idea

A fraction is in **lowest terms** when the top and bottom share no factor except 1. Divide both by their greatest common factor to get there in one step.



6 of 8 parts shaded

Two fractions are equivalent when they name the same amount of a whole, like $\frac{1}{2}$ and $\frac{2}{4}$: cutting each half into two pieces doubles the number of pieces and doubles the number you have, so nothing changes. Multiplying or dividing the top and bottom by the same number is the only move that keeps a fraction equal to itself.

Simplifying $\frac{6}{8}$

Both share the factor 2: $\dfrac{6 \div 2}{8 \div 2} = \dfrac{3}{4}$. The picture shows the same shaded amount.

Where it goes wrong

Adding the same number to the top and bottom. $\frac{1}{2}$ is not $\frac{2}{3}$ — adding 1 to each is not the same as cutting the pieces smaller. Only multiplying or dividing both parts by the same number keeps the value.

3.2 In Plain Terms & More Examples

In plain terms

A fraction is in lowest terms when the top and bottom share no common factor but 1. Divide both by the biggest factor they share.

Simplifying $\frac{8}{12}$

Both share 4: $\dfrac{8 \div 4}{12 \div 4} = \dfrac{2}{3}$.

Simplifying $\frac{9}{15}$

Both share 3: $\dfrac{9 \div 3}{15 \div 3} = \dfrac{3}{5}$.

3.3 Comparing Fractions

Same bottom, then compare tops

To compare $\frac{2}{3}$ and $\frac{3}{4}$, rewrite both with denominator 12: $\frac{8}{12}$ and $\frac{9}{12}$. Now the tops decide: $\frac{3}{4}$ is bigger.



8 of 12 parts shaded

Ordering $\frac{1}{2}, \frac{3}{5}, \frac{4}{10}$

Use tenths: $\frac{5}{10}, \frac{6}{10}, \frac{4}{10}$. From least to greatest: $\frac{4}{10}, \frac{1}{2}, \frac{3}{5}$.

Try it: which is larger?

Which is larger, $\frac{5}{8}$ or $\frac{7}{12}$?

Answer. Common denominator 24: $\frac{5}{8} = \frac{15}{24}$ and $\frac{7}{12} = \frac{14}{24}$. So $\mathbf{\frac{5}{8}}$ is larger.

3.4 Word Problems

Two pizzas

Ana ate $\frac{3}{8}$ of a pizza and Ben ate $\frac{2}{6}$ of a same-size pizza. $\frac{3}{8} = \frac{9}{24}$ and $\frac{2}{6} = \frac{8}{24}$, so Ana ate slightly more.

Try it: the garden

In a garden $\frac{6}{9}$ of the beds have tomatoes. Write that fraction in lowest terms.

Answer. The GCF of 6 and 9 is 3: $\frac{6\div3}{9\div3} = \mathbf{\frac{2}{3}}$.

3.5 Why It Works

Multiplying the numerator and denominator by the same number k is multiplying by $\frac{k}{k} = 1$, and multiplying by 1 changes nothing: $\frac{a}{b} \cdot \frac{k}{k} = \frac{ak}{bk}$. In a picture, $\frac{ak}{bk}$ is the same shaded region as $\frac{a}{b}$ with each piece cut into k thinner slices. Reducing to lowest terms runs the same idea backwards, dividing out every common factor until the greatest common factor of the top and bottom is 1.

Reading a proof

Do not read a proof the way you read a rule. Cover the last line, ask what the previous line forces, and only then uncover it. If a step feels like magic, that is the step to work by hand with a concrete number.

4 Adding Fractions

Same denominator, then different

4.1 Why Denominators Must Match

The Big Idea

You can only add pieces that are the **same size**. The denominator names the piece size, so it must match before you add the tops.



2 of 8 parts shaded

You can only add pieces that are the same size. Two fifths plus one fifth is three fifths — the pieces are fifths, so you count them. When the pieces are different sizes, like thirds and fourths, you first cut both into the same size (twelfths), and then the counting works again.

Finding $\frac{2}{8} + \frac{3}{8}$

The pieces are both eighths, so add the counts: $2 + 3 = 5$ eighths, or $\frac{5}{8}$.

Where it goes wrong

Adding the denominators: $\frac{1}{3} + \frac{1}{3}$ is $\frac{2}{3}$, not $\frac{2}{6}$. The bottom number names the piece size, and adding pieces does not change their size.

4.2 In Plain Terms & More Examples

In plain terms

When the denominators already match, just add or subtract the top numbers — the piece size (the bottom) stays the same.

Finding $\frac{3}{10} + \frac{4}{10}$

Same tenths, so add the tops: $3 + 4 = 7$, giving $\frac{7}{10}$.

Finding $\frac{5}{6} - \frac{2}{6}$

Same sixths, so subtract the tops: $5 - 2 = 3$, giving $\frac{3}{6} = \frac{1}{2}$.

4.3 Mixed Numbers

Whole parts and fraction parts

Add the wholes and the fractions separately, then regroup if the fraction part is 1 or more. Or convert everything to improper fractions first — both work.

Finding $2\frac{3}{4} + 1\frac{1}{2}$

Wholes: $2 + 1 = 3$. Fractions: $\frac{3}{4} + \frac{2}{4} = \frac{5}{4} = 1\frac{1}{4}$. Total $3 + 1\frac{1}{4} = 4\frac{1}{4}$.

Try it: subtract

Compute $3\frac{1}{5} - 1\frac{2}{5}$.

Answer. Convert: $\frac{16}{5} - \frac{7}{5} = \frac{9}{5} = \mathbf{1\frac{4}{5}}$. (Subtracting fraction parts first would need borrowing: $\frac{1}{5} < \frac{2}{5}$.)

4.4 Word Problems

Ribbon

Maya has $\frac{7}{8}$ m of ribbon and uses $\frac{3}{8}$ m. Left: $\frac{7}{8} - \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$ m.

Try it: the walk

Leo walked $\frac{2}{3}$ km before lunch and $\frac{5}{6}$ km after. How far did he walk in all?

Answer. Sixths: $\frac{4}{6} + \frac{5}{6} = \frac{9}{6} = \mathbf{1\frac{1}{2}}$ km.

4.5 Why It Works

A denominator is a unit, exactly like ”inches” or ”apples”. Three apples plus two apples is five apples, and $\frac{3}{5} + \frac{2}{5}$ is five fifths for the same reason: the unit ”fifth” is unchanged and only the count changes. When units differ we convert to a common unit first — inches to feet, or thirds and fourths to twelfths ($\frac{1}{3} = \frac{4}{12}$, $\frac{1}{4} = \frac{3}{12}$) — and then $\frac{4}{12} + \frac{3}{12} = \frac{7}{12}$ is just counting twelfths.

Reading a proof

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5 Decimals to Hundredths

Tenths, hundredths and lining up the point

5.1 Place Value After the Point

The Big Idea

The places keep dividing by ten past the decimal point: tenths, then hundredths. Lining up the points lines up the places.

A decimal is a fraction with a hidden denominator of 10, 100, or 1000. The digit right after the point counts tenths, the next counts hundredths, so 0.35 means 3 tenths and 5 hundredths — $\frac{35}{100}$. Money is the friendliest example: \$0.35 is 35 cents out of 100.

Adding $2.40 + 1.35$

Line up the points. Hundredths: $0 + 5 = 5$. Tenths: $4 + 3 = 7$. Wholes: $2 + 1 = 3$. The sum is 3.75.

Where it goes wrong

Reading 0.5 as smaller than 0.35 because $5 < 35$. Line the decimals up by place value: 0.50 versus 0.35 — fifty hundredths is more than thirty-five hundredths.

5.2 In Plain Terms & More Examples

In plain terms

Line up the decimal points so tenths sit under tenths and hundredths under hundredths, then add place by place.

Adding $3.25 + 1.50$

Hundredths $5 + 0 = 5$, tenths $2 + 5 = 7$, wholes $3 + 1 = 4$: the sum is 4.75.

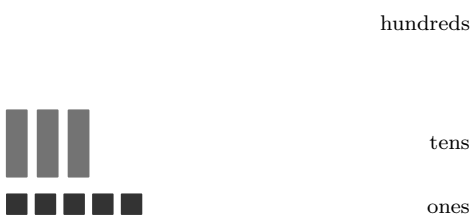
Adding $0.60 + 0.35$

Hundredths $0 + 5 = 5$, tenths $6 + 3 = 9$: the sum is 0.95.

5.3 Comparing and Rounding

Pad with zeros

To compare 0.5 and 0.35, write 0.50 and 0.35 — fifty hundredths beats thirty-five. Rounding to tenths looks at the hundredths digit: 5 or more rounds up.



Ordering $0.7, 0.68, 0.72$

As hundredths: $0.70, 0.68, 0.72$. Least to greatest: $0.68, 0.7, 0.72$.

Try it: round

Round 4.86 to the nearest tenth, and write $\frac{9}{100}$ as a decimal.

Answer. $4.86 \to \mathbf{4.9}$ (the 6 rounds the 8 up). $\frac{9}{100} = \mathbf{0.09}$ — a 0 holds the tenths place.

5.4 Word Problems

Change from a dollar

A pencil costs \$0.35. Paying with \$1.00: $1.00 - 0.35 = 0.65$, so the change is 65 cents.

Try it: two purchases

A notebook costs \$1.45 and a pen costs \$0.80. What is the total?

Answer. Line up the points: $1.45 + 0.80 = \$\mathbf{2.25}$.

5.5 Why It Works

Our number system gives each place a value ten times the one to its right: thousands, hundreds, tens, ones. Continuing the pattern to the right of the ones gives tenths, hundredths, thousandths — each place is a tenth of the last. So $0.35 = 3 \cdot \frac{1}{10} + 5 \cdot \frac{1}{100} = \frac{30}{100} + \frac{5}{100} = \frac{35}{100}$. Decimals are not a new kind of number; they are the place-value system extended below one.

Reading a proof

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6 Factors and Multiples

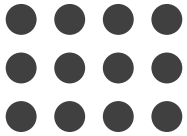
What divides evenly

6.1 Factors in Pairs

The Big Idea

Factors come in **pairs** that multiply to the number, which is why you only have to test up to

the square root — after that the pairs repeat.



3 rows of 4

A factor of 12 is a number that divides it exactly: $1, 2, 3, 4, 6, 12$. A multiple of 12 is 12 times something: $12, 24, 36, \ldots$ Factors are small and finite; multiples go on forever. A prime number has exactly two factors, itself and 1.

The factors of 12

1×12, 2×6, 3×4. So the factors are $1, 2, 3, 4, 6, 12$ — six of them. Each pair is one possible array.

Where it goes wrong

Mixing up the two words. Ask: is it smaller than the number (a factor, divides into it) or larger (a multiple, made from it)?

6.2 In Plain Terms & More Examples

In plain terms

A factor divides evenly with nothing left over; a multiple is what you get by counting up in steps of the number.

The factors of 18

1×18, 2×9, 3×6. So the factors are $1, 2, 3, 6, 9, 18$.

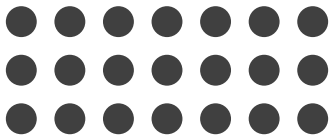
The first multiples of 4

Count up in fours: $4, 8, 12, 16, 20, \ldots$

6.3 Prime or Composite?

Test small primes

A number is **prime** if its only factors are 1 and itself. To test, try dividing by $2, 3, 5, 7$: if none divides it and the number is less than 121, it is prime.



3 rows of 7

Is 91 prime?

Not even; digits sum to 10 so not divisible by 3; does not end in 0 or 5; $91 \div 7 = 13$. So $91 = 7 \times 13$ is composite.

Try it: list the factors

List all the factors of 48, and decide whether 53 is prime.

Answer. Factor pairs of 48: $1 \cdot 48, 2 \cdot 24, 3 \cdot 16, 4 \cdot 12, 6 \cdot 8$ — factors $\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{6}, \mathbf{8}, \mathbf{12}, \mathbf{16}, \mathbf{24}, \mathbf{48}$. 53: not divisible by $2, 3, 5$ or 7, so **53** is prime.

6.4 Word Problems

Equal teams

36 players can form equal teams of $2, 3, 4, 6, 9, 12, 18$ or 36 — the factors of 36 (other than 1).

Try it: blinking lights

One light blinks every 6 seconds and another every 8 seconds. They blink together now. When do they next blink together?

Answer. The least common multiple of 6 and 8: multiples of 8 are $8, 16, 24$; 24 is the first that is also a multiple of 6. In **24** seconds.

6.5 Why It Works

Factors come in pairs because $a \times b = n$ means $b = n \div a$: every factor a has a partner $n \div a$. That is why you only need to test numbers up to the square root of n to find all its factors — once a

passes $\sqrt{n}$, its partner is smaller than $\sqrt{n}$ and was already found. For 36, testing 1 through 6 finds the pairs $(1, 36), (2, 18), (3, 12), (4, 9), (6, 6)$, and 6 pairs with itself because 36 is a perfect square.

Reading a proof

Do not read a proof the way you read a rule. Cover the last line, ask what the previous line forces, and only then uncover it. If a step feels like magic, that is the step to work by hand with a concrete number.

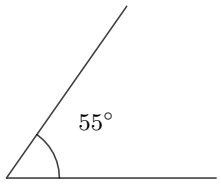
7 Angles

Measuring and classifying turns

7.1 Naming Angles

The Big Idea

An angle measures a **turn**. A quarter turn is 90° (a right angle), and a half turn is 180° (a straight angle).



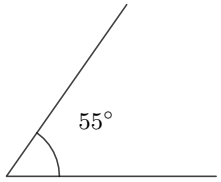
An angle measures how far one ray has turned away from another. A full turn is 360°, so a quarter turn — the corner of a square — is 90°, a right angle. Angles smaller than a right angle are acute; larger ones (but less than a straight line) are obtuse.

The three names

Less than 90° is **acute**. Exactly 90° is **right**. Between 90° and 180° is **obtuse**.

This angle

It opens less than a quarter turn, and it measures 55°, so it is acute.



Where it goes wrong

Measuring from the wrong zero on the protractor. Every protractor has two scales; use the one that starts at 0 on the ray you began with, and sanity-check: does the angle look smaller or bigger than a right angle?

7.2 In Plain Terms & More Examples

In plain terms

An angle measures how far something turns. Compare it to a right angle (90°): smaller is acute, bigger (up to 180°) is obtuse.

A 120° angle

It is more than 90° but less than 180°, so it is obtuse.

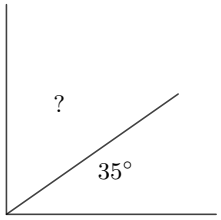
A 30° angle

It is less than 90°, so it is acute.

7.3 Adding and Splitting Angles

Angles add

When a ray splits an angle into two parts, the parts add to the whole. On a straight line the parts add to 180°.



The missing piece

A right angle is split into $35°$ and another angle. The other is $90° - 35° = 55°$.

Try it: on a line

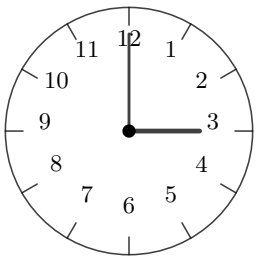
Two angles make a straight line. One is $112°$. What is the other, and is it acute or obtuse?

Answer. $180° - 112° = \mathbf{68°}$, which is less than $90°$: **acute**.

7.4 Word Problems

The clock

At 3 o'clock the hands make a quarter turn: $\frac{1}{4}$ of $360° = 90°$, a right angle.



Try it: turning

A skateboarder does a "540". How many full turns is that, and how much more?

Answer. $540 = 360 + 180$: **one full turn and a half turn** — she ends up facing backwards.

7.5 Why It Works

Why 360? A circle can be split into any number of equal turns, and 360 was chosen because it divides so evenly: by $2, 3, 4, 5, 6, 8, 9, 10, 12$ and more. That is why halves, thirds, quarters and sixths of a turn are all whole numbers of degrees. Angles add like lengths: two angles that share a ray combine into one whose measure is the sum, which is the fact behind "the angles on a straight line add to $180°$" — half a turn.

Reading a proof

Do not read a proof the way you read a rule. Cover the last line, ask what the previous line forces, and only then uncover it. If a step feels like magic, that is the step to work by hand with a concrete number.

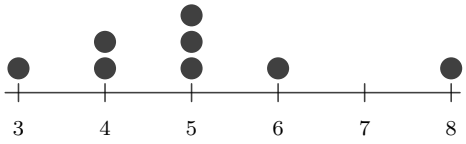
8 Line Plots

Showing data with dots

8.1 Reading a Line Plot

The Big Idea

Each dot is one measurement. Stacks show repeats, so the tallest stack is the most common value and the spread left to right is the **range**.



A line plot puts an X above each value every time it occurs, so the picture shows both what values there are and how often. Tall stacks are common values; gaps are values nobody had. You can read the total count by counting the X marks and the most common value by finding the tallest stack.

Finding the range

The largest value is 8 and the smallest is 3, so the range is $8 - 3 = 5$.

Where it goes wrong

Forgetting that each X is one data point. To find "how many measured more than 2", count the X marks above every value bigger than 2, not the number of columns.

8.2 In Plain Terms & More Examples

In plain terms

Each dot is one measurement. The tallest stack is the most common value (the mode), and the distance from smallest to largest is the range.

Reading a plot

For the values $2, 3, 3, 3, 5$, the value 3 appears most, so the mode is 3.

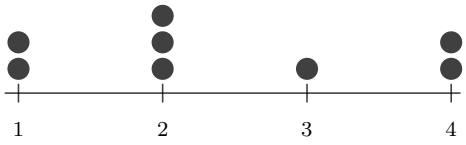
Its range

The largest is 5 and the smallest is 2, so the range is $5 - 2 = 3$.

8.3 Reading Fraction Line Plots

Fractions on the axis

Measurements are often fractions ($\frac{1}{4}$ inch, $\frac{1}{2}$ inch, $\frac{3}{4}$ inch). The plot works the same way: each X is one measurement stacked above its value.



How many altogether?

A plot shows 3 X's above $\frac{1}{4}$, 5 above $\frac{1}{2}$, and 2 above $\frac{3}{4}$. There are $3+5+2 = 10$ measurements, and the most common is $\frac{1}{2}$.

Try it: reading the plot

In that plot, how many measurements are greater than $\frac{1}{4}$? What is the difference between the largest and smallest values?

Answer. Greater than $\frac{1}{4}$: $5 + 2 = \mathbf{7}$. Largest minus smallest: $\frac{3}{4} - \frac{1}{4} = \mathbf{\frac{1}{2}}$.

8.4 Making a Plot

From a list

Shoe sizes: $3, 4, 4, 5, 3, 4, 6, 5$. Sort them: $3, 3, 4, 4, 4, 5, 5, 6$. Put two X's above 3, three above 4, two above 5, one above 6.

Try it: total

Using the shoe-size plot above, how many students are there, and which size is the mode (most common)?

Answer. Count the X's: $2 + 3 + 2 + 1 = \mathbf{8}$ students. The tallest stack is size **4**.

8.5 Why It Works

A line plot is a picture of a frequency table. Counting the marks in a stack recovers the frequency, and the marks to the right of a value count the data above it, so every question about the data can be answered by counting marks. Because the values sit on a number line, distances mean something: the plot shows the spread (largest minus smallest) at a glance, which a bar chart of categories cannot.

Reading a proof

Do not read a proof the way you read a rule. Cover the last line, ask what the previous line forces, and only then uncover it. If a step feels like magic, that is the step to work by hand with a concrete number.