MathCastle

STATE COMPETITION

Sprint Round

Problems 1–30

in the format of the MATHCOUNTS competition

Name _____

School _____

Chapter _____

INSTRUCTIONS

1. This round has 30 problems. You have 40 minutes to work on all of them.
2. No calculators, books or other aids. Work on scratch paper.
3. Write only your final answer in the blank in the right-hand column. Answers must be complete, legible and simplified, with fractions in lowest terms.
4. If you finish before time is called, use the remaining time to check your answers.

Total Correct	Scorer's Initials

DO NOT BEGIN UNTIL YOU ARE TOLD TO DO SO.

MathCastle practice paper. Original problems in the contest format; not affiliated with or endorsed by the contest organisers.

1. The three interior angles of a triangle are in the ratio $2 : 3 : 4$. At each vertex, an exterior angle is formed by one side of the triangle and the extension of the other side through that vertex. What is the degree measure of the largest of the triangle's three exterior angles?

1. _____

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2. A convex polygon has exactly three obtuse interior angles (angles greater than $90°$ and less than $180°$); each of its other interior angles measures at most $90°$. What is the largest possible number of sides of the polygon?

2. _____

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3. Each of the six digits $1, 2, 3, 4, 5, 6$ is used exactly once to write two three-digit numbers. What is the smallest possible positive difference between the two numbers?

3. _____

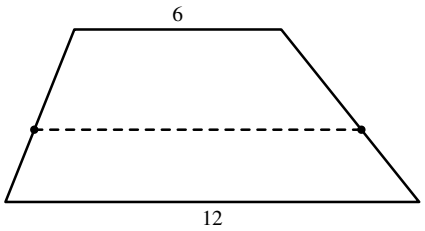
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4. A box contains 6 blue balls and at least two red balls, and nothing else. Two balls are drawn at random at the same time. The probability that both balls are red is exactly equal to the probability that one ball is red and the other is blue. How many balls are in the box?

4. _____

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5. A trapezoid has parallel sides of lengths 6 and 12. A segment parallel to these two sides, with one endpoint on each leg, divides the trapezoid into two pieces of equal area. What is the length of this segment? Express your answer in simplest radical form.



5. _____

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6. Let $N = 1 \cdot 3 \cdot 5 \cdot 7 \cdots 197 \cdot 199$ be the product of all the odd integers from 1 to 199. What is the largest integer k for which 5^k divides N?

6. _____

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7. A fair six-sided die is rolled again and again, and the rolling stops as soon as a 6 comes up for the first time. What is the probability that every roll before that 6 was an even number? (If the very first roll is a 6, this counts as a success, since there were no earlier rolls.) Express your answer as a common fraction.

7. _____

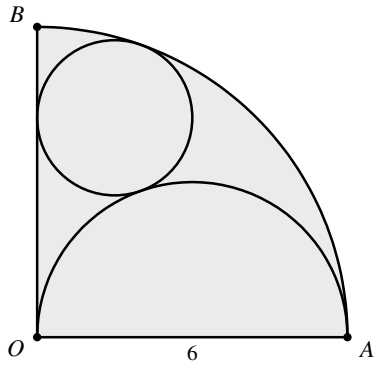
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8. Find the sum of all positive integers $n < 1000$ for which $n^2 + (n+1)^2$ is a perfect square.

8. _____

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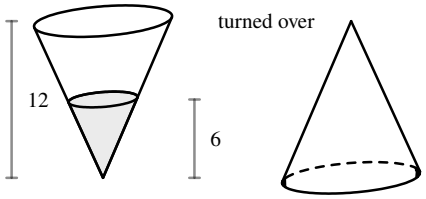
9. Point O is the centre of a circle of radius 6, and OA and OB are radii perpendicular to each other; the quarter-disc bounded by OA, OB and arc AB is shaded. Inside the quarter-disc, a semicircle is drawn with diameter OA (bulging toward B). A small circle lies inside the quarter-disc, outside the semicircle, and is tangent to segment OB, to arc AB and to the semicircle. What is the radius of the small circle? Express your answer as a common fraction.



9. _____

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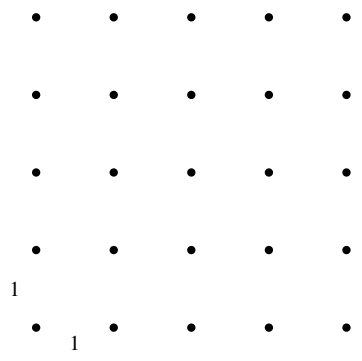
10. A closed container has the shape of a right circular cone of height 12. It is held with its vertex pointing straight down, and water fills it to a depth of 6, measured up from the vertex. The container is then turned over so that it rests on its circular base with the vertex pointing up. What is the depth of the water now, measured up from the base? Express your answer in simplest radical form.



10. _____

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11. Twenty-five dots are arranged in a square array of 5 rows and 5 columns, with neighbouring dots in each row and each column exactly 1 unit apart. Two different dots are chosen at random, with every pair of dots equally likely. What is the probability that the straight-line distance between the two dots is a whole number? Express your answer as a common fraction.



11. _____

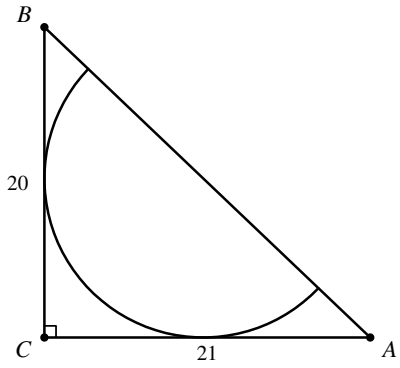
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12. For some integers n (positive, negative or zero), the absolute value $|n^2 - 16n + 39|$ is a prime number. What is the sum of all such integers n?

12. _____

MathCastle practice paper. Original problems in the contest format; not affiliated with or endorsed by the contest organisers.

13. Right triangle ABC has its right angle at C, with legs $CA = 21$ and $CB = 20$. A semicircle lies inside the triangle; its diameter lies along the hypotenuse $\overline{AB}$, and its curved arc is tangent to both legs $\overline{CA}$ and $\overline{CB}$. What is the radius of the semicircle? Express your answer as a common fraction.



13. _____

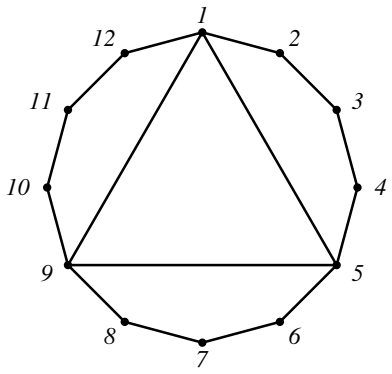
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14. A cyclist leaves town A at 8:00 a.m. and rides along a straight road toward town B. A car leaves A at 9:00 a.m. on the same road and overtakes the cyclist at 10:00 a.m. The car reaches B at 11:00 a.m., immediately turns around, and drives back toward A until it meets the cyclist. Each of them moves at their own constant speed the whole time. How many minutes after 11:00 a.m. does the car meet the cyclist?

14. _____

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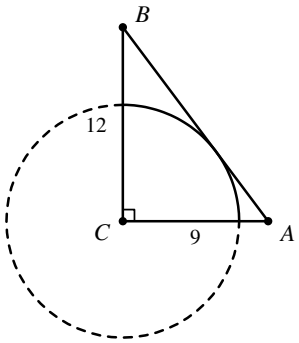
15. The vertices of a convex 12-gon are labelled 1 through 12 in order around it. The three diagonals joining vertices 1 and 5, vertices 5 and 9, and vertices 9 and 1 are coloured red. How many of the other diagonals of the 12-gon cross at least one red diagonal at a point in the interior of the 12-gon?



15. _____

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16. Right triangle ABC has its right angle at C, with $CA = 9$ and $CB = 12$. A circle is drawn with center C, just large enough to touch the hypotenuse $\overline{AB}$ (so the circle is tangent to $\overline{AB}$). What is the area of the part of triangle ABC that lies outside the circle? Express your answer in terms of π.



16. _____

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17. For how many positive integers n does the base-2 representation of n have exactly twice as many digits as the base-6 representation of n? For example, $n = 10$ counts, since $10 = 1010_2$ has 4 digits and $10 = 14_6$ has 2 digits.

17. _____

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18. How many four-digit positive integers are written using exactly two different digits? For example, 3113, 7000 and 2522 each count, but 5555 and 1231 do not.

18. _____

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19. In triangle ABC, $AB = 6$ and $AC = 9$. The bisector of $\angle BAC$ meets side $\overline{BC}$ at point D, and M is the midpoint of side $\overline{BC}$. Triangle ADM has area 2. What is the area of triangle ABC?

19. _____

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20. For a real number x, let $\lfloor x \rfloor$ be the greatest integer less than or equal to x. What is the sum of all positive integers n for which $\left\lfloor \frac{n}{2} \right\rfloor + \left\lfloor \frac{n}{3} \right\rfloor + \left\lfloor \frac{n}{5} \right\rfloor = n - 1$?

20. _____

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21. Three fair six-sided dice, one red, one white and one blue, are rolled. What is the probability that the number on one of the dice equals the sum of the numbers on the other two dice? Express your answer as a common fraction.

21. _____

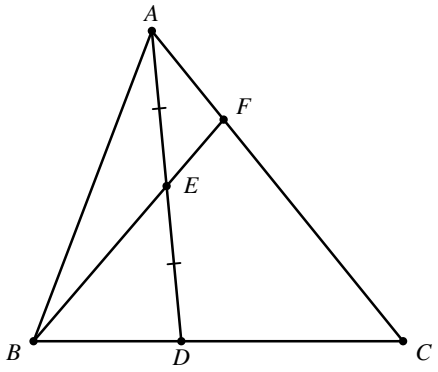
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22. How many four-digit positive integers are multiples of 11 and have digits that add up to 13?

22. _____

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23. In triangle ABC, point D lies on side $\overline{BC}$ with $BD : DC = 2 : 3$. Point E is the midpoint of segment $\overline{AD}$. The line through B and E meets side $\overline{AC}$ at point F. What is the value of $\frac{AF}{FC}$? Express your answer as a common fraction.



23. _____

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24. Each of the nine squares of a 3×3 grid is coloured red or blue, each colour with probability $\frac{1}{2}$, independently. The grid contains four 2×2 blocks of four squares (one in each corner of the grid, all sharing the center square). What is the probability that none of these four 2×2 blocks is all one colour? Express your answer as a common fraction.

24. _____

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25. Jug A holds 6 litres of a juice-and-water mixture that is 20% juice, and jug B holds 4 litres of a mixture that is 70% juice. At the same moment, x litres are poured from jug A into jug B and x litres are poured from jug B into jug A (each pour is taken from the jug's original, unmixed contents), and then both jugs are stirred. Afterwards the two jugs contain exactly the same percentage of juice. What is x? Express your answer as a common fraction.

25. _____

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26. The three altitudes of a triangle have lengths 3, 4 and 6. What is the area of the triangle? Express your answer in simplest radical form.

26. _____

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27. For how many positive integers $n \leq 200$ is the number of positive divisors of n that are multiples of 3 an odd number? (For example, $n = 12$ counts: its divisors that are multiples of 3 are 3, 6 and 12, which is three of them.)

27. _____

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28. Call a positive integer squareful if it is divisible by some perfect square greater than 1. For example, 12 and 27 are squareful, but 30 is not. What is the smallest positive integer N such that all four of the consecutive integers N, $N+1$, $N+2$ and $N+3$ are squareful?

28. _____

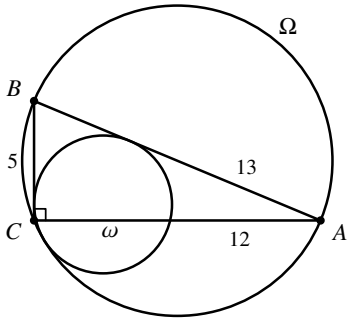
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29. Twelve people sit in the twelve chairs around a round table, one person per chair. They all stand up and sit down again, one person per chair, and each person either sits back down in his or her own chair or in one of the two chairs immediately next to it. Two outcomes are different if some person ends up in a different chair. How many different outcomes are possible? (Everyone returning to his or her own chair counts as one outcome.)

29. _____

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30. Right triangle ABC has its right angle at C, with legs $BC = 5$ and $CA = 12$ and hypotenuse $AB = 13$. Let Ω be the circle through A, B and C. A circle ω lies inside Ω and inside the angle $\angle ABC$; it is tangent to side $\overline{BC}$, tangent to side $\overline{AB}$, and tangent to Ω (touching it from the inside). The circle ω is not inside the triangle; it bulges out past side $\overline{CA}$. What is the radius of ω? Express your answer as a common fraction.



30. _____

Answer Key — MathCounts State Sprint Practice Contest 3

1. 140

2. 6

3. 47

4. 19

5. $3\sqrt{10}$

6. 25

7. $\frac{1}{4}$

8. 838

9. $\frac{3}{2}$

10. $12 - 6\sqrt[3]{7}$

11. $\frac{9}{25}$

12. 16

13. $\frac{420}{41}$

14. 20

15. 36

16. $54 - \frac{324\pi}{25}$

17. 78

18. 567

19. 20

20. 165

21. $\frac{5}{24}$

22. 21

23. $\frac{2}{5}$

24. $\frac{161}{256}$

25. $\frac{12}{5}$

26. $\frac{16\sqrt{15}}{5}$

27. 8

28. 242

29. 324

30. $\frac{26}{9}$

MathCastle practice paper. Original problems in the contest format; not affiliated with or endorsed by the contest organisers.

Solutions — MathCounts State Sprint Practice Contest 3

1. Answer: 140**. The idea.** An exterior angle and the interior angle at the same vertex add to 180°, so the largest exterior angle sits next to the *smallest* interior angle, not the largest one.

Work it out. Write the angles as $2k, 3k, 4k$. Then $9k = 180$, so $k = 20$ and the angles are $40°, 60°, 80°$. The exterior angles are $180 - 40 = 140$, $180 - 60 = 120$ and $180 - 80 = 100$ degrees. The largest is 140°.

Check. The three exterior angles of any triangle add to 360°: $140 + 120 + 100 = 360$. Also, an exterior angle equals the sum of the two remote interior angles: $60 + 80 = 140$.

2. Answer: 6**. The idea.** Interior angles are awkward to control because their sum grows with the number of sides, but the *exterior* angles of a convex polygon always add to exactly 360°. A non-obtuse interior angle ($\le 90°$) forces a big exterior angle ($\ge 90°$), and there is room for only a few of those.

Work it out. Each obtuse interior angle has an exterior angle strictly between 0° and 90°, so the three obtuse corners contribute a positive amount to the 360°. Every other corner contributes at least 90°. If there were four or more non-obtuse corners, the exterior angles would total at least $4 \cdot 90°$ plus something positive, which is more than 360°. So there are at most 3 non-obtuse corners, and at most $3 + 3 = 6$ sides.

Check. Six sides really happen: a hexagon whose interior angles alternate 90°, 150°, 90°, 150°, 90°, 150° has exterior angles 90°, 30°, 90°, 30°, 90°, 30°, which add to 360°; with sides alternating between two lengths it has threefold symmetry and closes up. Its angle sum $3 \cdot 90 + 3 \cdot 150 = 720 = 4 \cdot 180$ is correct for a hexagon. For a heptagon the angle sum 900° would need the three obtuse angles to total at least $900 - 4 \cdot 90 = 540°$, impossible since each is under 180°.

3. Answer: 47**. The idea.** The hundreds digits dominate, so they should differ by exactly 1. After that, the larger number should be as small as possible and the smaller number as large as possible: give the larger number the smallest leftover digits and the smaller number the largest ones.

Work it out. If the hundreds digits are $h+1$ and h, the difference is 100 plus the last two digits of the larger number minus the last two digits of the smaller number. For each choice of hundreds digits, write the smallest two-digit ending with two of the leftover digits on the larger number and the largest ending with the other two on the smaller. Hundreds $2, 1$ (leftovers $3, 4, 5, 6$): $234 - 165 = 69$. Hundreds $3, 2$: $314 - 265 = 49$. Hundreds $4, 3$: $412 - 365 = 47$. Hundreds $5, 4$: $512 - 463 = 49$. Hundreds $6, 5$: $612 - 543 = 69$. The smallest is 47.

Check. Hundreds digits that differ by 2 or more already give a difference above 100, so they cannot win. With hundreds 4 and 3, the larger number's ending 12 is the smallest ending possible from any digits and 65 is the largest, so no arrangement at all can beat $100 + 12 - 65 = 47$. A computer search over all 720 arrangements also gives 47.

4. Answer: 19**. The idea.** Both probabilities have the same denominator (the number of pairs of balls), so they are equal exactly when the *numbers* of pairs are equal. Compare the counts, not the fractions.

MathCastle practice paper. Original problems in the contest format; not affiliated with or endorsed by the contest organisers.

Work it out. Let there be r red balls. The number of red–red pairs is $\binom{r}{2} = \frac{r(r-1)}{2}$, and the number of red–blue pairs is $6r$. Setting them equal: $\frac{r(r-1)}{2} = 6r$. Since $r > 0$, divide by r: $r - 1 = 12$, so $r = 13$. The box holds $13 + 6 = 19$ balls.

Check. With 19 balls there are $\binom{19}{2} = 171$ pairs; $\binom{13}{2} = 78$ are red–red and $13 \cdot 6 = 78$ are red–blue. Both probabilities are $\frac{78}{171}$.

5. Answer: $3\sqrt{10}$**. The idea.** The height of the trapezoid is not given and is not needed. Extend the two legs until they meet: every segment parallel to the bases then cuts off a triangle similar to the others, and the areas of similar triangles go as the *square* of the parallel side.

Work it out. Let the legs meet at P. The triangle cut off by a parallel segment of length x has area kx^2 for one fixed constant k. So the triangle on the side of length 6 has area $36k$, the triangle on the side of length 12 has area $144k$, and the trapezoid has area $144k - 36k = 108k$. The halving segment of length x must cut off a triangle of area $36k + 54k = 90k$, so $x^2 = 90$ and $x = \sqrt{90} = 3\sqrt{10}$.

Check. $3\sqrt{10} \approx 9.49$ lies between 6 and 12. It is longer than the midline $\frac{6+12}{2} = 9$, as it should be: the half of the trapezoid next to the long base is wider, so the halving line has to sit closer to the long base. In general the answer is $\sqrt{\frac{a^2+b^2}{2}}$, here $\sqrt{\frac{36+144}{2}} = \sqrt{90}$.

6. Answer: 25**. The idea.** Count factors of 5 the way you would for a factorial, one layer at a time (multiples of 5, then of 25, then of 125), but count only the *odd* multiples, since the even numbers are not in the product.

Work it out. The odd multiples of 5 up to 199 are $5 \cdot 1, 5 \cdot 3, \ldots, 5 \cdot 39$: that is 20 numbers, each giving one factor of 5. The odd multiples of 25 are $25, 75, 125, 175$: 4 numbers, each giving a second factor. The only odd multiple of 125 is 125, giving a third factor. So $k = 20 + 4 + 1 = 25$.

Check. Another route: $N = \dfrac{200!}{2 \cdot 4 \cdot 6 \cdots 200} = \dfrac{200!}{2^{100} \cdot 100!}$. The factorial counts give 200! exactly $40 + 8 + 1 = 49$ factors of 5 and 100! exactly $20 + 4 = 24$, and $49 - 24 = 25$.

7. Answer: $\frac{1}{4}$**. The idea.** Rolls of 2 and 4 change nothing: after one of them the situation is exactly as it was. The story is decided by the first roll that is *not* a 2 or a 4, and that roll is equally likely to be any of $1, 3, 5, 6$.

Work it out. If the first such roll is a 6, every earlier roll was a 2 or a 4, so the event happens. If it is a 1, 3 or 5, an odd roll came before the first 6, so the event fails. The four values are equally likely, so the probability is $\frac{1}{4}$. As a sum: the event happens with exactly k rolls of 2 or 4 followed by a 6 with probability $\left(\frac{1}{3}\right)^k \cdot \frac{1}{6}$, and $\frac{1}{6}\left(1 + \frac{1}{3} + \frac{1}{9} + \cdots\right) = \frac{1}{6} \cdot \frac{3}{2} = \frac{1}{4}$.

Check. The tempting answer $\frac{1}{2}$ ("each earlier roll is even half the time") ignores that the earlier rolls are known not to be 6, and that a long run of rolls is unlikely to be all even. The first roll alone already gives success with probability $\frac{1}{6}$ and failure with probability $\frac{1}{2}$; the remaining $\frac{1}{3}$ restarts the game, so $p = \frac{1}{6} + \frac{1}{3}p$, giving $p = \frac{1}{4}$.

8. Answer: 838**. The idea.** Such an n gives a right triangle with legs n and $n + 1$. Consecutive integers share no factor, so the triple is primitive, and every primitive triple has legs $p^2 - q^2$ and $2pq$. Requiring those two legs to differ by 1 turns into a Pell equation, whose solutions come one after another.

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Work it out. We need $(p^2 - q^2) - 2pq = \pm 1$, that is $(p-q)^2 - 2q^2 = \pm 1$. With $x = p - q$ and $y = q$, the solutions of $x^2 - 2y^2 = \pm 1$ are $(1,1), (3,2), (7,5), (17,12), (41,29), \ldots$ (each next pair is $(x + 2y, x + y)$). Then $p = x + y$, $q = y$ gives $(p,q) = (2,1), (5,2), (12,5), (29,12), (70,29)$ and the legs $\{3,4\}$, $\{20,21\}$, $\{119,120\}$, $\{696,697\}$, $\{4059,4060\}$. The last is too big, so $n = 3, 20, 119, 696$ and the sum is $3 + 20 + 119 + 696 = 838$.

Check. $3^2+4^2 = 5^2$, $20^2+21^2 = 29^2$, $119^2+120^2 = 169^2$ and $696^2+697^2 = 970225 = 985^2$. The values also follow the pattern $n_{\text{next}} = 6n - n_{\text{prev}} + 2$: $6 \cdot 20 - 3 + 2 = 119$ and $6 \cdot 119 - 20 + 2 = 696$.

9. Answer: $\frac{3}{2}$**. The idea.** Tangency of circles is a statement about the distance between centres: internally tangent means the distance is the *difference* of the radii, externally tangent means the *sum*. Two such equations share the same unknown height, and subtracting them cancels it.

Work it out. Put $O = (0,0)$, $A = (6,0)$, $B = (0,6)$; the semicircle has centre $(3,0)$ and radius 3. A circle of radius r tangent to OB has centre (r, y). Tangent inside the big circle: $r^2 + y^2 = (6-r)^2$. Tangent outside the semicircle: $(r-3)^2 + y^2 = (r+3)^2$. Subtracting the second from the first: $r^2 - (r-3)^2 = (6-r)^2 - (r+3)^2$, that is $6r - 9 = 27 - 18r$, so $24r = 36$ and $r = \frac{3}{2}$.

Check. Then $y^2 = (6 - \frac{3}{2})^2 - (\frac{3}{2})^2 = \frac{81}{4} - \frac{9}{4} = 18$, so the centre is $(\frac{3}{2}, 3\sqrt{2})$. Its distance to $(3,0)$ is $\sqrt{\frac{9}{4} + 18} = \frac{9}{2} = 3 + \frac{3}{2}$, and to O is $\frac{9}{2} = 6 - \frac{3}{2}$. The circle touches OB at height $3\sqrt{2} \approx 4.24 < 6$, so everything lies inside the quarter-disc.

10. Answer: $12 - 6\sqrt[3]{7}$**. The idea.** Volumes of similar cones go as the *cube* of their heights. Vertex down, the water is a small cone; vertex up, the *air* is a small cone at the top. So work with the air, not the water.

Work it out. The water cone has half the height of the container, so it holds $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ of the volume. After turning over, the air fills the other $\frac{7}{8}$, and it forms a cone at the top similar to the container. Its height h satisfies $\left(\frac{h}{12}\right)^3 = \frac{7}{8}$, so $h = 12 \cdot \frac{\sqrt[3]{7}}{2} = 6\sqrt[3]{7}$. The water depth is $12 - 6\sqrt[3]{7}$.

Check. $\sqrt[3]{7} \approx 1.913$, so the depth is about 0.52: a thin layer, which is reasonable because the wide base region holds a lot of volume. Indeed the air cone of height $6\sqrt[3]{7}$ has volume fraction $\frac{7 \cdot 216}{1728} = \frac{7}{8}$, leaving $\frac{1}{8}$ for the water.

11. Answer: $\frac{9}{25}$**. The idea.** A distance $\sqrt{a^2 + b^2}$ (with a, b the horizontal and vertical gaps, each at most 4) is a whole number when the dots share a row or column, and also in one sneaky case: the 3–4–5 right triangle.

Work it out. There are $\binom{25}{2} = 300$ pairs. Same row: each of the 5 rows has $\binom{5}{2} = 10$ pairs, so 50; same column: another 50. Otherwise we need $a^2 + b^2$ to be a perfect square with $1 \le a, b \le 4$; checking, only $\{a,b\} = \{3,4\}$ works (distance 5). Gaps of 3 across and 4 up: the left dot's column can be the first or second (2 choices) and the rows are forced (1 choice), and the slope can go up or down, giving $2 \cdot 2 = 4$ pairs. Gaps of 4 across and 3 up: another 4. Total $50 + 50 + 8 = 108$, and $\frac{108}{300} = \frac{9}{25}$.

Check. The possible values of $a^2 + b^2$ with $1 \le a \le b \le 4$ are $2, 5, 10, 17, 8, 13, 20, 18, 25, 32$; only 25 is a perfect square, confirming that the 3–4–5 pairs are the only diagonal ones.

12. Answer: 16**. The idea.** Factor: $n^2 - 16n + 39 = (n-3)(n-13)$. A product of two integers can be $\pm$ a prime only if one factor is ± 1, which leaves just four candidates to test.

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Work it out. If $|n-3|=1$, then $n=4$ or $n=2$. If $|n-13|=1$, then $n=14$ or $n=12$. Test each: $n=4$: $1\cdot(-9)$, absolute value 9, not prime. $n=12$: $9\cdot(-1)$, again 9. $n=2$: $(-1)(-11)=11$, prime. $n=14$: $11\cdot 1=11$, prime. So $n=2$ and $n=14$, with sum 16.

Check. The two factors always differ by 10, so when one of them is ±1 the other is ±9 or ±11. Only 11 is prime, which is why exactly two of the four candidates survive. They are symmetric about the vertex $n=8$, so their sum is $2\cdot 8=16$.

13. Answer: $\frac{420}{41}$**. The idea.** Let O be the centre of the semicircle; it lies on $\overline{AB}$. The radii to the two points of tangency are perpendicular to the legs, so they are heights of triangles OCA and OCB. Those two triangles fill up triangle ABC, so their areas add up to its area.

Work it out. Call the radius r. The area of ABC is $\frac{1}{2}\cdot 21\cdot 20=210$. Triangle OCA has base $CA=21$ and height r (the distance from O to $\overline{CA}$), and triangle OCB has base $CB=20$ and height r. So $\frac{1}{2}\cdot 21r+\frac{1}{2}\cdot 20r=210$, which gives $\frac{41}{2}r=210$ and $r=\frac{420}{41}$.

Check. Put C at the origin, $A=(21,0)$ and $B=(0,20)$. A point at distance r from both legs is (r,r), and it lies on the hypotenuse $\frac{x}{21}+\frac{y}{20}=1$ when $r\left(\frac{1}{21}+\frac{1}{20}\right)=1$, that is $r=\frac{420}{41}\approx 10.24$. The diameter fits on the hypotenuse, since O is more than r from each of A and B (about 14.9 and 14.1).

14. Answer: 20**. The idea.** No distances are given, but none are needed: the overtaking tells you the ratio of the speeds, and everything else can be measured in "cyclist-hours" of road.

Work it out. Let the cyclist cover 1 unit of road per hour. At 10:00 both have covered the same distance: the cyclist in 2 hours and the car in 1 hour, so the car's speed is 2 units per hour. The car drives from 9:00 to 11:00, so B is 4 units from A. At 11:00 the cyclist is 3 units from A, so 1 unit short of B. Now they approach each other at a combined $1+2=3$ units per hour, closing the 1-unit gap in $\frac{1}{3}$ hour, which is 20 minutes.

Check. At 11:20 the cyclist has ridden $\frac{10}{3}$ hours, so is $\frac{10}{3}$ units from A. The car has driven back for $\frac{1}{3}$ hour at speed 2, so is $4-\frac{2}{3}=\frac{10}{3}$ units from A. They agree.

15. Answer: 36**. The idea.** Count the complement. Two diagonals of a convex polygon cross inside it exactly when their endpoints alternate around the polygon. So a diagonal avoids every red diagonal exactly when both of its endpoints lie on one of the three arcs $1,2,3,4,5$ or $5,6,7,8,9$ or $9,10,11,12,1$ cut off by the red triangle.

Work it out. The 12-gon has $\frac{12\cdot 9}{2}=54$ diagonals, so 51 are not red. Each arc has 5 vertices, which form $\binom{5}{2}=10$ segments; 4 of them are sides of the 12-gon and one is red, leaving 5 non-red diagonals that cross nothing red. Two arcs share only a single vertex, so these $3\cdot 5=15$ diagonals are all different. Every other non-red diagonal has an endpoint strictly inside one arc and the other endpoint outside that arc, so it crosses the red diagonal closing off that arc. The answer is $51-15=36$.

Check. Count directly with inclusion–exclusion. Diagonal 1–5 is crossed by the $3\cdot 7=21$ diagonals from $\{2,3,4\}$ to $\{6,\dots,12\}$, and likewise for the other two red diagonals: 63. A diagonal crosses two red diagonals exactly when it joins two of the groups $\{2,3,4\}$, $\{6,7,8\}$, $\{10,11,12\}$: $3\cdot 9=27$ of them, and none crosses all three. So $63-27=36$.

16. Answer: $54-\frac{324\pi}{25}$**. The idea.** A circle centred at C and tangent to line AB has radius equal to

the distance from C to AB, which is the altitude to the hypotenuse. The real question is what the circle looks like inside the triangle: if the radius is shorter than both legs, the circle never reaches past A or B, so the part of the disc inside the triangle is exactly the quarter-disc cut out by the right angle at C.

Work it out. The hypotenuse is $AB = \sqrt{9^2 + 12^2} = 15$ and the area is $\frac{1}{2} \cdot 9 \cdot 12 = 54$. Computing the area as $\frac{1}{2} \cdot AB \cdot h$ gives $h = \frac{2 \cdot 54}{15} = \frac{36}{5}$, so the radius is $r = \frac{36}{5} = 7.2$.

Since $7.2 < 9$ and $7.2 < 12$, the circle crosses each leg at an interior point, and the only point of the circle on or beyond $\overline{AB}$ is the point of tangency. So inside the triangle the disc is the quarter-disc between the two legs, with area $\frac{1}{4}\pi r^2 = \frac{1}{4} \cdot \frac{1296}{25}\pi = \frac{324}{25}\pi$.

The part of the triangle outside the circle has area $54 - \frac{324}{25}\pi$.

Check. Numerically, $\frac{324}{25}\pi \approx 40.72$, leaving about 13.28: two thin corner regions near A and B, which is plausible since the quarter-disc fills most of the triangle. The answer is $54 - \frac{324\pi}{25}$.

17. Answer: 78**. The idea.** A number has exactly k digits in base 6 when it lies in the block $6^{k-1} \le n \le 6^k - 1$, and exactly $2k$ digits in base 2 when it lies in the block $2^{2k-1} \le n \le 2^{2k} - 1$. For each k, count the overlap of the two blocks. Because 6^{k-1} grows faster than 4^k, the blocks eventually stop overlapping.

Work it out. $k = 1$*:* $1 \le n \le 5$ and $2 \le n \le 3$, giving $n = 2, 3$: 2 numbers.

$k = 2$*:* $6 \le n \le 35$ and $8 \le n \le 15$, giving $8 \le n \le 15$: 8 numbers.

$k = 3$*:* $36 \le n \le 215$ and $32 \le n \le 63$, giving $36 \le n \le 63$: 28 numbers.

$k = 4$*:* $216 \le n \le 1295$ and $128 \le n \le 255$, giving $216 \le n \le 255$: 40 numbers.

$k \ge 5$*:* the base-6 block starts at 6^{k-1}, but the base-2 block ends at $4^k - 1$. At $k = 5$, $6^4 = 1296 > 1023 = 4^5 - 1$, and each step multiplies the first by 6 and the second (roughly) by only 4, so there is never any overlap again.

The total is $2 + 8 + 28 + 40 = 78$.

Check. Spot-check the edges of the $k = 4$ range: $216 = 1000_6$ has 4 base-6 digits and $216 = 11011000_2$ has 8 binary digits, while $256 = 100000000_2$ already has 9 binary digits. And $35 = 55_6$ has only 6 binary digits (100011_2), not 4, so it is correctly excluded. The answer is 78.

18. Answer: 567**. The idea.** Pick the pair of digits first, then count the four-digit strings that use both of them. The digit 0 needs separate treatment because it cannot lead.

Work it out. *Two nonzero digits.* There are $\binom{9}{2} = 36$ pairs. Each of the four places can hold either digit, which gives $2^4 = 16$ strings, but 2 of these use only one of the digits. That leaves 14 per pair, for $36 \cdot 14 = 504$ numbers.

A nonzero digit a together with 0*.* The first digit must be a. Each of the other three places is a or 0, and at least one of them must be 0: $2^3 - 1 = 7$ ways. With 9 choices of a that is 63 numbers.

The total is $504 + 63 = 567$.

Check. Count by complement. Of the 9000 four-digit numbers, 9 use one digit and $9 \cdot 9 \cdot 8 \cdot 7 = 4536$ use four different digits. For exactly three different digits, choose the two places holding the repeated digit ($\binom{4}{2} = 6$ ways) and fill the three "slots" with different digits so that the leading digit is not 0: $720 - 72 = 648$ ways, for $6 \cdot 648 = 3888$. Then $9000 - 9 - 4536 - 3888 = 567$. The answer is 567.

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19. Answer: 20**. The idea.** Triangles with the same apex A and bases along line BC have the same height, so their areas are in the ratio of their bases. The angle bisector theorem tells us where D is, so DM is a known fraction of BC.

Work it out. By the angle bisector theorem, $BD : DC = AB : AC = 6 : 9 = 2 : 3$, so $BD = \frac{2}{5}BC$. The midpoint has $BM = \frac{1}{2}BC$. Since $\frac{2}{5} < \frac{1}{2}$, D lies between B and M, and $DM = \left(\frac{1}{2} - \frac{2}{5}\right) BC = \frac{1}{10}BC$. So triangle ADM has $\frac{1}{10}$ of the area of triangle ABC, and the area of ABC is $10 \cdot 2 = 20$.

Check. With total area 20: triangle ABD has area $\frac{2}{5} \cdot 20 = 8$, triangle ADM has 2, and triangle AMC has 10. Then $ABD + ADM = 10 = AMC$, as it should be for the median, and $ABD : ADC = 8 : 12 = 2 : 3$, as it should be for the bisector. Such a triangle really exists: two sides 6 and 9 can enclose any area up to $\frac{1}{2} \cdot 6 \cdot 9 = 27$, and $20 < 27$. The area is 20.

20. Answer: 165**. The idea.** Without the floors the left side would be $\frac{n}{2} + \frac{n}{3} + \frac{n}{5} = \frac{31n}{30}$. Each floor removes a fractional part, and those fractional parts are $\frac{r_2}{2}$, $\frac{r_3}{3}$, $\frac{r_5}{5}$, where r_2, r_3, r_5 are the remainders of n on division by 2, 3, 5. So the equation says the three fractional parts add up to exactly $1 + \frac{n}{30}$. Since they can never add up to 2, n must be small, and the remainders then determine everything.

Work it out. The equation becomes $\frac{r_2}{2} + \frac{r_3}{3} + \frac{r_5}{5} = 1 + \frac{n}{30}$, or $15r_2 + 10r_3 + 6r_5 = 30 + n$. The left side is at most $15 + 20 + 24 = 59$, so $n \le 29$. For each remainder triple, n is then forced to be $15r_2 + 10r_3 + 6r_5 - 30$, and we need this to be positive (it is automatically below 30; and by the Chinese remainder theorem, $15r_2 + 10r_3 + 6r_5$ always leaves the remainders r_2, r_3, r_5 itself, so the forced n really has those remainders). Go through the pairs (r_2, r_3):

$(0,0)$: $6r_5 - 30 \le -6$, none. $(1,0)$: $6r_5 - 15 > 0$ for $r_5 = 3, 4$: $n = 3, 9$. $(0,1)$: $6r_5 - 20 > 0$ for $r_5 = 4$: $n = 4$. $(1,1)$: $6r_5 - 5 > 0$ for $r_5 = 1, 2, 3, 4$: $n = 1, 7, 13, 19$. $(0,2)$: $6r_5 - 10 > 0$ for $r_5 = 2, 3, 4$: $n = 2, 8, 14$. $(1,2)$: $6r_5 + 5$ for all $r_5 = 0, \ldots, 4$: $n = 5, 11, 17, 23, 29$.

That is 15 values, with sum $12 + 4 + 40 + 24 + 85 = 165$.

Check. $n = 29$: 14+9+5 = 28. $n = 6$ (not on the list): 3+2+1 = 6, not 5. $n = 30$: 15+10+6 = 31, already more than $n - 1$, and from then on the left side only pulls further ahead. The fourteen integers from 1 to 29 that fail are $6, 10, 12, 15, 16, 18, 20, 21, 22, 24, 25, 26, 27, 28$, summing to 270, and $435 - 270 = 165$. The answer is 165.

21. Answer: $\frac{5}{24}$**. The idea.** Decide which die shows the sum. Once that is fixed, the other two dice can be anything whose total is at most 6. Then make sure no roll is counted for two different choices of "sum die".

Work it out. Say the red die is the sum. The white and blue dice show (a, b) with $a + b \le 6$: there are $1 + 2 + 3 + 4 + 5 = 15$ such ordered pairs (sums 2 through 6), and each forces the red value. The same 15 outcomes arise with white or blue as the sum die, for 45 in all.

Could one roll be counted twice? That would need, say, $r = w + b$ and $w = r + b$ at once; adding gives $b = 0$, impossible. So the 45 outcomes are all different, and the probability is $\frac{45}{216} = \frac{5}{24}$.

Check. A small list for the red die being largest: $(2; 1, 1)$, $(3; 1, 2)$, $(3; 2, 1)$, $(4; 1, 3)$, $(4; 2, 2)$, $(4; 3, 1)$, which already accounts for $1 + 2 + 3$ of the 15. The probability is $\frac{5}{24}$.

22. Answer: 21**. The idea.** Write the number as $\overline{abcd}$. It is a multiple of 11 exactly when the alternating sum $(a + c) - (b + d)$ is a multiple of 11. We also know $(a + c) + (b + d) = 13$, which

is odd, so the difference is odd too: it cannot be 0, and it lies between -13 and 13. So it must be 11 or -11.

Work it out. *Difference* $+11$*:* $a+c=12$ and $b+d=1$. With digits, $a+c=12$ has the 7 solutions $(3,9),(4,8),\ldots,(9,3)$, and $b+d=1$ has 2 solutions. That gives 14 numbers.

Difference -11*:* $a+c=1$ and $b+d=12$. Since $a\geq 1$, the only choice is $a=1$, $c=0$; and $b+d=12$ has 7 solutions. That gives 7 numbers.

In total there are $14+7=21$ numbers.

Check. Examples: 3190 has digit sum 13 and $3190=11\cdot 290$; 1309 has digit sum 13 and $1309=11\cdot 119$. The count is 21.

23. Answer: $\frac{2}{5}$. The idea. Use mass points: put weights at the vertices so that each marked point is the balance point of the segment it lies on. Then the ratio on $\overline{AC}$ can be read off from the weights at A and C.

Work it out. For D to balance B and C with $BD:DC=2:3$, give B weight 3 and C weight 2, so D carries 5. For E to be the midpoint of $\overline{AD}$, A must carry the same weight as D, namely 5. Now E is the balance point of the whole system, so the line BE passes through the balance point of A and C alone, which is F. Balancing A (weight 5) and C (weight 2) gives $AF:FC=2:5$, so $\frac{AF}{FC}=\frac{2}{5}$.

Check. Coordinates: $B=(0,0)$, $C=(5,0)$, $D=(2,0)$, $A=(0,5)$. Then $E=(1,\frac{5}{2})$ and line BE is $y=\frac{5}{2}x$. Line AC is $y=5-x$. They meet at $x=\frac{10}{7}$, which is $\frac{10/7}{5}=\frac{2}{7}$ of the way from A to C. So $AF:FC=2:5$.

24. Answer: $\frac{161}{256}$. The idea. Count the colourings where at least one block is a single colour, using inclusion–exclusion over the four blocks. The key detail is that overlapping blocks share the center square, so if two blocks are both one-coloured they must be the same colour.

Work it out. There are $2^9=512$ colourings. Call the blocks TL, TR, BL, BR.

One block. A given block is one colour in $2\cdot 2^5=64$ colourings; four blocks give 256.

Two blocks. Side-by-side blocks (like TL, TR) share 2 squares and cover 6, so both are one colour in $2\cdot 2^3=16$ colourings; there are 4 such pairs, giving 64. Diagonal blocks (like TL, BR) share only the center and cover 7 squares: $2\cdot 2^2=8$ colourings; there are 2 such pairs, giving 16. Total 80.

Three blocks. Any three cover all squares except one corner: $2\cdot 2=4$ colourings each, 4 triples, 16 in all.

Four blocks. The whole grid is one colour: 2.

So $256-80+16-2=190$ colourings have a one-colour block, and $512-190=322$ do not. The probability is $\frac{322}{512}=\frac{161}{256}$.

Check. The "bad" count must at least be the number with a monochrome TL block, 64, and at most 256; 190 is in range. A computer run over all 512 grids confirms 322 good colourings.

25. Answer: $\frac{12}{5}$. The idea. No juice is created or lost, and after the exchange each jug again holds its original volume. If the two jugs end with the same percentage of juice, that percentage must be the percentage of all 10 litres taken together. So there is only one jug to worry about.

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Work it out. Altogether there are $0.2 \cdot 6 + 0.7 \cdot 4 = 1.2 + 2.8 = 4$ litres of juice in 10 litres, which is 40%. After the exchange jug A holds $6 - x$ litres of its own 20% mixture and x litres of the 70% mixture, still 6 litres in all. For it to be 40% juice it must contain 2.4 litres of juice: $0.2(6 - x) + 0.7x = 2.4$, so $1.2 + 0.5x = 2.4$ and $x = \frac{12}{5}$.

Check. Jug B then holds $4 - \frac{12}{5} = \frac{8}{5}$ litres of 70% mixture and $\frac{12}{5}$ litres of 20% mixture: $0.7 \cdot 1.6 + 0.2 \cdot 2.4 = 1.12 + 0.48 = 1.6$ litres of juice out of 4, which is 40% too. (In general the same reasoning gives $x = \frac{V_A V_B}{V_A + V_B} = \frac{6 \cdot 4}{10}$, whatever the two percentages are, as long as they differ.) The answer is $\frac{12}{5}$.

26. Answer: $\frac{16\sqrt{15}}{5}$**. The idea.** If the area is K, the side that goes with altitude h has length $\frac{2K}{h}$. So the sides are proportional to the *reciprocals* of the altitudes, which fixes the shape of the triangle; Heron's formula then gives a second expression for the area, and comparing the two fixes the size.

Work it out. The sides are $\frac{2K}{3}$, $\frac{2K}{4}$ and $\frac{2K}{6}$, in the ratio $\frac{1}{3} : \frac{1}{4} : \frac{1}{6} = 4 : 3 : 2$. Write them as $4t$, $3t$, $2t$ with $4t = \frac{2K}{3}$, that is, $K = 6t$. (These do form a triangle, since $2 + 3 > 4$.) Heron's formula with $s = \frac{9t}{2}$ gives $K = \sqrt{\frac{9t}{2} \cdot \frac{t}{2} \cdot \frac{3t}{2} \cdot \frac{5t}{2}} = \frac{t^2\sqrt{135}}{4} = \frac{3\sqrt{15}}{4}t^2$. Setting $\frac{3\sqrt{15}}{4}t^2 = 6t$ gives $t = \frac{8}{\sqrt{15}}$, so $K = 6t = \frac{48}{\sqrt{15}} = \frac{16\sqrt{15}}{5}$.

Check. Numerically $t \approx 2.066$, the sides are about 8.26, 6.20 and 4.13, and $K \approx 12.39$. Then $\frac{2K}{8.26} \approx 3.00$, $\frac{2K}{6.20} \approx 4.00$ and $\frac{2K}{4.13} \approx 6.00$, the given altitudes. (Taking the sides proportional to $3 : 4 : 6$ instead is the classic mistake.) The area is $\frac{16\sqrt{15}}{5}$.

27. Answer: 8**. The idea.** If 3 divides n, the divisors of n that are multiples of 3 match up one-to-one with the divisors of $\frac{n}{3}$: send d to $\frac{d}{3}$. And a number has an odd number of divisors exactly when it is a perfect square, because divisors pair up as d and $\frac{m}{d}$ except when $d = \sqrt{m}$.

Work it out. If 3 does not divide n, then n has no divisors that are multiples of 3, and 0 is even. If 3 divides n, the count equals the number of divisors of $\frac{n}{3}$, which is odd exactly when $\frac{n}{3}$ is a perfect square. So n works exactly when $n = 3k^2$ for a positive integer k. We need $3k^2 \le 200$, that is, $k^2 \le 66$, so $k = 1, 2, \ldots, 8$. The numbers are $3, 12, 27, 48, 75, 108, 147, 192$: there are 8.

Check. $n = 27$: the divisors that are multiples of 3 are $3, 9, 27$, three of them. $n = 18$ (not of the form $3k^2$): they are $3, 6, 9, 18$, four of them. And $n = 72 = 3 \cdot 24$ fails because 24 is not a square: its divisors that are multiples of 3 are $3, 6, 9, 12, 18, 24, 36, 72$, eight of them. The answer is 8.

28. Answer: 242**. The idea.** A number is squareful exactly when it is divisible by p^2 for some prime p. Among four consecutive integers only one is a multiple of 4, and no odd prime square $(9, 25, 49, \ldots)$ can divide two of them, because they differ by at most 3. So the other three numbers need three *different* odd primes, and since only 3 and 5 are below 7, one of the four numbers must be a multiple of p^2 for a prime $p \ge 7$. That leaves only a few places to look.

Work it out. The multiples of $49, 121, 169, \ldots$ up to 245 are $49, 98, 121, 147, 169, 196, 242, 245$. Check each for a run of four squareful numbers containing it, using nearby numbers that are *not* squareful (products of distinct primes):

49: 47 and $51 = 3 \cdot 17$ are not squareful, so every run of four containing 49 fails (48, 49, 50 is the most we get).

98: 97 and 101 are prime, and every run of four containing 98 contains one of them.

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121: $119 = 7 \cdot 17$ and $122 = 2 \cdot 61$, so 121 has non-squareful neighbours on both sides.

147: $146 = 2 \cdot 73$ and 149 is prime; the runs containing 147 contain one of $145 = 5 \cdot 29$, 146 or 149.

169: 167 is prime and $170 = 2 \cdot 5 \cdot 17$.

196: $195 = 3 \cdot 5 \cdot 13$ and 197 is prime.

$242 = 2 \cdot 11^2$: 241 is prime, but $242 = 2 \cdot 11^2$, $243 = 3^5$, $244 = 2^2 \cdot 61$ and $245 = 5 \cdot 7^2$ are all squareful.

Any run starting below 242 would contain one of the listed multiples below 242 (or 242 itself, with 241 inside the run), and all of those failed. So $N = 242$.

Check. The run $242, 243, 244, 245$ uses the squares $121, 9, 4, 49$: one multiple of 4 and three different odd primes $11, 3, 7$, just as the idea predicts. A computer search of every N below 242 agrees that none works.

29. Answer: 324**. The idea.** Show that, apart from two special outcomes in which everyone moves the same way around the table, every outcome is made of people who stay put and pairs of neighbours who swap chairs. Counting sets of non-overlapping neighbouring pairs around a circle is a Fibonacci-style count.

Work it out. Number the chairs 1 to 12 around the table and suppose the person from chair i moves clockwise to chair $i+1$. Someone must fill chair i: the person from $i+1$ or the person from $i-1$. If it is the person from $i+1$, the two neighbours have swapped. If not, the person from $i-1$ also moved clockwise into chair i; then chair $i-1$ must be filled by the person from $i-2$ (the person from i has gone to $i+1$), and so on all the way around. So either every clockwise move is part of a swap, or everybody moves clockwise. The same holds counterclockwise.

Everyone rotates: 2 outcomes (all clockwise or all counterclockwise).

Swaps and stays: choose a set of non-overlapping neighbouring pairs of chairs to swap. For a row of m chairs, the number a_m of such sets satisfies $a_m = a_{m-1} + a_{m-2}$ (the last chair either stays or swaps with the one before it), with $a_1 = 1$, $a_2 = 2$: $1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233$, so $a_{10} = 89$ and $a_{12} = 233$. Around the circle, either chairs 12 and 1 do not swap with each other (a row of 12: 233 sets) or they do (the other 10 chairs form a row: 89 sets). That is $233 + 89 = 322$ outcomes.

Altogether $322 + 2 = 324$.

Check. For 4 people the same reasoning gives $a_4 + a_2 + 2 = 5 + 2 + 2 = 9$: the identity, 4 single swaps, 2 double swaps and 2 rotations. Listing all outcomes for 4 people confirms 9. For 12 people the answer is 324.

30. Answer: $\frac{26}{9}$**. The idea.** A circle tangent to both sides of $\angle ABC$ has its center on the bisector of that angle, so its center and radius depend on just one number. The circumcircle of a right triangle has the hypotenuse as a diameter, so its center and radius are known. Internal tangency says the distance between the centers equals the difference of the radii; that one equation fixes the unknown.

Work it out. Put $C = (0, 0)$, $B = (0, 5)$ and $A = (12, 0)$. The center of Ω is the midpoint of $\overline{AB}$, $O = \left(6, \frac{5}{2}\right)$, and its radius is $\frac{13}{2}$.

From B, side $\overline{BC}$ points in direction $(0,-1)$ and side $\overline{BA}$ in direction $\left(\frac{12}{13},-\frac{5}{13}\right)$. The angle bisector points along their sum $\left(\frac{12}{13},-\frac{18}{13}\right)$, i.e. along $(2,-3)$. So the center of ω is $P=(2s,5-3s)$ for some $s>0$, and its radius is its distance to line BC (the y-axis), which is $\rho=2s$.

Internal tangency: $OP=\frac{13}{2}-\rho$, so $(2s-6)^2+\left(\frac{5}{2}-3s\right)^2=\left(\frac{13}{2}-2s\right)^2$. Expanding, $4s^2-24s+36+9s^2-15s+\frac{25}{4}=\frac{169}{4}-26s+4s^2$, which simplifies to $9s^2-13s=0$. Since $s>0$, $s=\frac{13}{9}$, and the radius is $\rho=\frac{26}{9}$.

Check. The center is $P=\left(\frac{26}{9},\frac{2}{3}\right)$. Its distance to line AB $(5x+12y=60)$ is $\frac{60-\frac{130}{9}-8}{13}=\frac{338/9}{13}=\frac{26}{9}$, equal to the radius as it should be. And $OP^2=\left(\frac{28}{9}\right)^2+\left(\frac{11}{6}\right)^2=\frac{3136}{324}+\frac{1089}{324}=\frac{4225}{324}=\left(\frac{65}{18}\right)^2$, while $\frac{13}{2}-\frac{26}{9}=\frac{65}{18}$, exactly as internal tangency requires. The center is above line CA by less than the radius ($\frac{2}{3}<\frac{26}{9}$), so ω does bulge past $\overline{CA}$. The radius is $\frac{26}{9}$ (it is larger than the inradius 2, as a circle reaching out to Ω must be).

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